A Case Against a Four Percent Inflation Target*

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Abstract

We reformulate standard New Keynesian models to include heterogeneity in prices stickiness suggested by micro-evidence on prices and positive trend inflation. An increase in trend inflation leads to richer inflation dynamics and heterogeneity in price mark-ups. These changes shrink the region in which the model is determinate significantly. When trend inflation is 4 percent, the determinacy region in the model is almost non-existent, cautioning against such a policy as a means to avoid the zero bound in the future, and pointing to the costs that high inflation may have had in the past.

Keywords: Trend inflation, determinacy, sticky prices, New Keynesian, Taylor Rules. JEL classification: E52, E61, E66, C14, C18

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1 Introduction

This paper adds both positive trend inflation and heterogeneity in the degree of price stickiness into an otherwise standard New Keynesian model and use it to study the determinacy of interest rate rules. Indeterminacy here refers to the property of rational expectations models whereby a single value for fundamental shocks like technology, tastes, can, under some conditions, be consistent with multiple possible values for (in this context) inflation. In the NK model indeterminacy is undesirable, because it opens up the possibility for self-fulfilling disturbances to inflation expectations, which, through the normal channels induced by price stickiness, (for example, the costs of relative price distortion), are costly. Our interest in determinacy derives from two prior strands of work, one normative, the other positive/empirical.

The normative literature on determinacy of interest rate rules followed less formal work on the desirability and comparative performance of interest rate rules. This is exemplified by the early work by Federal Reserve Board researchers - for example, in the volume edited by Bryant, Hooper and Mann (1993) - and the famous paper by Taylor (1993). Formal analyses of determinacy in NK models has its antecedent in studies of the earlier generation of rational expectations, monetary models. For example, writing when the relative merits of money versus interest rate rules was in play, Sargent and Wallace (1975) show that if interest rates are 'pegged' at a level invariant to conditions in the macro economy, there are multiple possible *price levels* consistent with given fundamentals under rational expectations. Later, McCallum (1981) points out that an interest rate rule could be designed to mimic any desired path of the money stock - since with agents on their money demand curves these were duals - thus pinning down the price level.

Rigorous assessment of the determinacy of the *inflation rate* in the dynamic, optimising, New Keynesian rational expectations model is first presented by Woodford (2003). The model is the simplest, closed-economy, one-distortion only (i.e. sticky prices) version of the NK model, and a central bank pursuing a variant of the Taylor Rule, in which the interest rate responds to terms in the current inflation rate and the gap between actual and potential output. The focus is on the necessity for interest rate rules to be specified in terms of endogenous variables, to generate determinacy, and on the necessity/sufficiency, or lack of it, of the so called 'Taylor Principle', which asserts that interest rates ought to respond more than one for one to fluctuations in the gap between inflation and its target value. Woodford concludes that the Taylor Principle is relevant in the NK model but the condition is different. The new condition is that "at least in the long-run, nominal interest rates should rise by more than the increase in the inflation rate (Woodford (2003, p. 254)).[1]

A related, positive, empirical monetary policy literature focus on the question of whether central banks has in the past followed interest rate rules that satisfy appropriate determinacy conditions. For example, Clarida, Gali and Gertler (2000) estimate policy rules for the Fed and find coefficients that suggest policy that would have generated indeterminacy prior to Volcker but not after his tenure. This view is confirmed by Lubik and Schorfheide (2004) who present full-model estimates. Orphanides (2001) suggests that the inadequately inflation-responsive policy in the 1960s and 1970s may have been due to inaccurate information about the output-gap fed into an otherwise sound policy rule. Lubik and Matthes (2016) argue that policy leading to indeterminacy is also due in part to inadequate information about model structure.

New light on both of these strands of work - normative and positive -

[1] Other papers that study determinay using NK models include Bernanke and Woodford (1997), Bullard and Mitra (2007) and Bullard and Schaling (2009). Bernanke and Woodford (1997) consider determinacy under rules that involved a feedback from inflation forecasts rather than actual inflation (a practice some central banks described themselves as following). Bullard and Mitra (2007) consider rules intertial rules with terms in lagged interest rates (which matched features of estimated central bank policy rules); and determinacy in the open economy NK model is studied by Bullard and Schaling (2009).

on indeterminacy was shed by Ascari and coauthors (see, for example, Ascari and Ropele (2008) and Ascari and Sbordone (2014)). He and others develop a modification of the standard New Keynesian model, which is typically approximated around a zero inflation steady state, allowing approximation around non-zero steady states. (See also Bakhshi, Burriel-Llombart, Khan and Rudolf (2003) and Kiley (2007)). Ascari and coauthors find that the range of parameters for which the model generated determinate rational expectations equilibria for inflation - the property that a single value for fundamental shocks maps into a single value for inflation - is narrowed as trend inflation increased. For example, with positive inflation, it was no longer sufficient, even in the otherwise simplest NK model, to have the inflation response coefficient greater than one.

Ascari's work was important for two reasons. First, it enumerates a possible, offsetting cost to consider, against the suggestion - by, for example Blanchard, Mauro and Dell'Ariccia (2014) and Ball (2014), that the inflation targets of central banks should be raised to around 4% to combat the zero bound to interest rates. This suggestion has followed from the several years spent at the effective lower bound to central bank interest rates by the US, UK, Japan and Eurozone, and the prospect of very low equilibrium real interest rates (implying, via the Fisher relation, a correspondingly low rest point for nominal rates) for the foreseeable future. Higher inflation targets would provide more room for cuts in nominal interest rates in response to future recessions. Second, Ascari's work speaks to the empirical determinacy literature and carries the implication that it is more likely that high and volatile inflation in the post WW2-pre-Greenpan era was due to indeterminacy, since it revealed that the determinacy condition which actual, realised central bank policy needed to have met was more stringent than would be guessed from the less realistic version of the model linearized around the zero steady state.

Our paper revisits the impact of positive trend inflation on the determinacy region in a model with many sectors with different degrees of price stickiness. We find that a realistic calibration of the degree of heterogeneity greatly amplifies the extent to which trend inflation narrows the determinacy region for monetary policy rules. The implication that flows from this is that much more emphasis on the normative and positive implications of the trend inflation work should be placed than has been hitherto.

Modifying the NK model to accommodate heterogeneity in price stickiness is not merely an academic exercise: it is an important feature of the micro evidence on prices (see Klenow and Malin (2011) for a recent survey: also Alvarez et al. (2006) and Hall, Walsh and Yates (2000)). Moreover, it has been shown that adding heterogeneity in price stickiness significantly improves the empirical performance of the standard model, both at the macro and micro levels. For example, Kara (2015) shows that two disturbing problems of the standard, (i.e. homogenous price stickiness) New Keynesian model disappear when heterogeneity in price stickiness is introduced. First, the model requires large price shocks to explain inflation dynamics (see Chari, Kehoe and McGrattan (2009)) and, second, firm-level pricing in the model is inconsistent with that in reality (see Bils, Klenow and Malin (2012)).

To introduce heterogeneity to the standard model, we employ the Multiple Calvo (MC) approach, as in Carvalho (2006) and Kara (2015). In the MC model there are many sectors, each with a different Calvo-style contract. Sectors face different probabilities of price adjustment. The share of each sector (which has a distinct expected contract duration) in the MC model is calibrated according to the Bils and Klenow (2004) dataset. We reformulate the MC model to include a positive trend inflation rate, analogously to the way Ascari and Sbordone (2014) introduce trend inflation into the representative firm version of the model. When sectors face the same probability of price adjustment, the model reduces to the Ascari and Sbordone (2014) model; when our model has identical probabilities of price adjustment and zero steady-state inflation, the model collapses further to the standard NK model.

To convey the intuition behind our results, we need to explain i) why trend inflation aggravates the determinacy problem and ii) why heterogeneity in price stickiness magnifies this effect.

Let us start by briefly explaining how allowing for positive trend inflation affects the price-setting process of firms. A main implication of higher trend inflation in the model is the same as that of higher price stickiness in the sense that, just like with increases in price stickiness, higher trend inflation dampens the effects of the business cycle on inflation. This is true for two reasons. First, since prices are sticky, the increased inflation target makes the price setting process more forward-looking in the sense that firms put more weight on the future. Second, with higher trend inflation, resetting firms increase their prices well above the average price level because they need to take into account of the fact that the pace at which their relative price will be eroded is greater, should they find themselves unable to change prices in the future. Increased reset prices means that these firms face lower demand for their products, reducing their expenditure share in the economy. Both of these reasons imply that inflation becomes less sensitive to changes in output.

Trend inflation affects steady-state price mark-ups and output too. This is a direct consequence of the fact that firms increase their prices more aggressively, as trend inflation increases, which leads to higher price mark-ups and, in turn, lower steady-state output. Therefore, a permanent increase in inflation leads to permanently lower output . To put it differently, as emphasised in Ascari and Ropele (2009), once we allow for the variations in the trend level of inflation in the NK model, we can see that the slope of the Phillips curve is negative.

These implications of trend inflation significantly affect the size of the region in which the model is determinate. As Woodford (2003) notes, to achieve determinacy, in the long-run, the nominal interest rate should increase by more than the increase in inflation. We assume that the central bank sets monetary policy according to a Taylor rule under which the nominal interest rate adjusts to react to changes in inflation and the output gap. Given these and the negative effect of a higher inflation target on long-run (or steady-state) output, the long-run nominal interest rate would not change as much as it would when the inflation target is low. As a result, when the inflation target is higher, if the central bank cares about output, it needs to react more aggressively to changes in inflation.

Now let us turn to the implications of trend inflation on the heterogeneous price-stickiness, MC model - the novel analysis in our paper - and the intuition for why introducing heterogeneity in price stickiness magnifies the impact of trend inflation on the determinacy region. Incorporating positive trend inflation into the MC leads to richer inflation dynamics. In the standard case with zero trend inflation, inflation in a sector in the MC depends on inflation in that sector, marginal costs as well as relative prices. Trend inflation introduces another channel through which sectors affect each other. Sectoral inflation depends also on expected aggregate inflation.

As in the standard, (common price-stickiness) model, allowing for positive trend inflation makes firms more forward-looking, resulting in resetting firms making larger price adjustments. The magnitude of adjustments depends on the degree of price stickiness. Sectors with sticky prices make larger adjustments than sectors with relatively flexible prices. The gap between the adjustments of relatively flexible and stickier price firms becomes larger with trend inflation.

An interesting and important implication of this feature of the MC model is that steady-state price mark-ups are different across sectors when trend inflation is positive. Without trend inflation, they are the same. Sector-specific mark-ups increases with the degree of price rigidity. As a result, steady-state output is lower in sticky-price sectors and decreases further with an increase in inflation target. In the MC model, there are longer-term contracts than in the Calvo model, making the slope of the Phillips curve in the steady-state even more negative than the corresponding Calvo model.

This more negative slope of the Phillips curve in the MC model greatly amplifies the degree to which increments to trend inflation shrink the determinacy region. For example, at the 4 percent inflation target, in the MC model, the region in which equilibrium is determinate is very small, implying that the model is indeterminate for a wide range of parameter values. In the corresponding standard model, at the 4 percent target, the determinacy region is much larger than that in the MC model and is not too different from the standard case with zero trend inflation. In the standard model, when the assumed target is 4 percent, indeterminacy becomes a problem only when the degree of price stickiness is significantly greater than that suggested by the micro-evidence. Recapping on the implications of our analysis once more: moving to a 4% inflation target to avoid the zero bound in the future looks much more likely to generate indeterminacy when one adds realistic heterogeneity in price stickiness into the model. And from a historical perspective, noting that the average inflation rate in the pre-inflation targeting era was greater than 4%, our model suggests that it was much more likely that indeterminacy explained excessive fluctuations in inflation than was apparent beforehand.

The remainder of the paper is organised as follows. Section 2 presents the model and discusses the calibration of model parameters. Section 3 examines the implications of heterogeneity in price stickiness on the determinacy region. In this section, we compare the results from the standard Calvo model with those from a simple two sector MC and a ten sector MC that is calibrated according to the Bils and Klenow (2004) dataset. In this section, we also discuss how trend inflation affects short-run inflation dynamics and long-run properties of the MC. Finally, Section 4 concludes the paper.

2 The Model

The model presented here incorporates heterogeneity in price stickiness into the model in Ascari and Sbordone (2014) model using the Multiple Calvo (MC) approach, as in Kara (2015). The model in Ascari and Sbordone (2014) is standard New Keynesian model, with one important exception. Ascari and Sbordone (2014) assumes that steady-state inflation can be positive. We first present the equations describing price setting in the MC and then the remaining equations which are identical to those in Ascari and Sbordone (2014) with logarithmic consumption utility. Finally, we discuss inflation dynamics implied by the model.

2.1 Multiple Calvo (MC) with trend Inflation

There is a continuum of monopolistically competitive, profit-maximising firms indexed by $f \epsilon [0, 1]$, each producing a differentiated good Y_f . Firms operate according to the following production function,

$$Y_{ft} = A_t N_{ft} \tag{1}$$

where are N denotes labour and A denotes labour-augmenting technology. These goods are then combined, according to the Dixit-Stiglitz techonology, to produce the final consumption Y_t. Y_t and the corresponding price index are given by

$$Y_t = \left[\int_0^1 P_{ft}^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{1-\varepsilon}} \tag{2}$$

$$P_t = \left[\int_0^1 P_{ft}^{1-\varepsilon} df \right]^{\frac{1}{1-\varepsilon}} \tag{3}$$

Under these assumptions, the demand for firm's i output is

$$Y_{ft} = p_{ft}^{-\varepsilon} Y_t \tag{4}$$

where $p_{ft} = P_{ft}^*/P_t$, P_{ft}^* is the price level set by firm f, P_t is the general price level and ε is the elasticity of substitution between different goods. To introduce heterogeneity in price stickiness to the model, the unit interval of firms is divided into segments which are interpreted as sectors. There are N sectors, $i = 1...N$. Within each sector i, there is a Calvo style contract. The share of sector i in the economy is α_i and the sector-specific Calvo hazard rate is denoted by $1 - \theta_i$. If we define the cumulative shares of sectors as $\bar{\alpha}_i = \sum_k^i \alpha_k$, where $k = 1...i$, $\bar{\alpha}_0 = 1$ and $\bar{\alpha}_N = 1$, then the interval for sector i is $[\bar{\alpha}_{i-1}, \bar{\alpha}_i]$. With these assumptions, the general price index (P_t) can be rewritten in terms of sectors as follows.

$$P_t = \left[\sum_{i=1}^{N} \int_{\bar{\alpha}_i - 1}^{\bar{\alpha}_i} P_{ft}^{1-\varepsilon} df \right]^{\frac{1}{1-\varepsilon}} \tag{5}$$

A firm in sector i in period t choose the optimal price $P_{fi,t}^*$ to maximise expected profits during the expected lifetime of the contract, subject to the demand curve and the production function. Solving the maximisation problem gives the following pricing rule for the firms in sector i

$$\frac{P_{it}^*}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{\psi_{it}}{\phi_{it}} \tag{6}$$

Where ψ_{it} and ϕ_{it} are defined as follows:

$$\psi_{it} = MC_t + \theta_i \beta E_t \pi_{t+1}^{\varepsilon} \psi_{it+1} \tag{7}$$

$$\phi_{it} = \theta_i \beta E_t \pi_{t+1}^{\varepsilon-1} \phi_{it+1} \tag{8}$$

Where $MC_t = w_t/A_t$ is the marginal cost, $w_t = W_t/P_t$ denotes real wages and W_t denotes nominal wages. $\pi_t = P_t/P_{t-1}$ is the inflation rate between t and $t - 1$. Log-linearising equations (6), (7) and (8) gives

$$\hat{p}^*_{it} = \hat{\psi}_{it} - \hat{\phi}_{it} \tag{9}$$

$$\hat{\psi}_{it} = (1 - \beta\theta_i\pi^{\varepsilon})\left(\hat{w}_t - \hat{A}_t\right) + \beta\theta_i\pi^{\varepsilon}\left(\varepsilon\hat{\pi}_{t+1} + \hat{\psi}_{it+1}\right) \tag{10}$$

$$\hat{\phi}_{it} = \beta\theta_i\pi^{\varepsilon-1}\left((\varepsilon - 1)\,\hat{\pi}_{t+1} + \hat{\phi}_{it+1}\right) \tag{11}$$

The aggregate price level in sector i is a weighted average of last period's aggregate price in sector i and the reset price for this period in that sector. The log-linearised sectoral (real) price ($\hat{p}_{it}$) is given by

$$\hat{p}_{it} = (1 - \theta_i)\,(\hat{p}^*_{it}) + \theta_i\pi^{\varepsilon-1}\,(\hat{p}_{it-1} - \hat{\pi}_t) \tag{12}$$

where $\hat{p}^*_{it}$ is the log-linearised (real) reset price in sector i. Related to this equation, price dispersion ($\hat{s}_{it}$) within each sector i is given by

$$\hat{s}_{it} = (1 - \theta_i)(\hat{p}^*_{it})^{-\varepsilon} + \theta_i\pi^{\varepsilon}_t\hat{s}_{it-1} \tag{13}$$

The aggregate price level in the economy is the weighted average of all ongoing prices in the economy. This relation of course implies that

$$\sum_{i=1}^{N}\alpha_i\hat{p}_{it} = 0 \tag{14}$$

$\hat{p}_{it}$ can be expressed as

$$\hat{p}_{it} = \hat{p}_{it-1} + \hat{\pi}_{it} - \hat{\pi}_t \tag{15}$$

These equations can also nest the model in Ascari and Sbordone (2014) by setting $N = 1$. The rest of the model is the same as in Ascari and Sbordone (2014) and the equations are repeated here for convenience. Output is given by the standard Euler condition:

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - (\hat{\imath}_t - E_t\hat{\pi}_{t+1})$$

This equation emerges, in the standard fashion, from log-linearizing the representative household's consumption Euler equation, and noting that since there is no capital formation in the model, consumption equals output at all times. Aggregate labour demand is given by

$$\hat{N}_t = \hat{s}_t + \hat{Y}_t - \hat{A}_t \tag{16}$$

where $\hat{s}_t = \sum_{i=1}^{N} \alpha_i \hat{s}_{it}$. The real wage is given by:

$$\hat{w}_t = \varphi \hat{N}_t + \sigma \hat{Y}_t \tag{17}$$

Using these two equations and the fact that $\widehat{MC}_t = \hat{w}_t - \hat{A}_t$, marginal cost can be expressed as follows:

$$\widehat{MC}_t = \varphi \hat{s}_t + (\varphi + \sigma) \hat{Y}_t - (1 + \varphi)\hat{A}_t \tag{18}$$

Monetary policy is modelled as following a Taylor rule:

$$\hat{\imath}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{Y}_t \tag{19}$$

where the $\phi-$coefficients are the parameters in front of the targeting variables.

2.2 Calibration

Following Ascari and Sbordone (2014), ε is set to 10. The discount factor assumed is $\beta = 0.99$. The share of each sector (or duration) (α_i) in the MC is calibrated based on the micro evidence provided by Bils and Klenow (BK) (2004) (BK-MC). To do so, we take data on the frequency of price changes reported by Bils and Klenow, who report this frequency for around 300 product categories, which covers 70% of the US CPI. Following Kara (2015), we aggregate up from their 300 sectors so that we have just 10 sectors with distinct price reset probabilities. The aggregation is performed by forming probability focal points in increments of 0.1 percentage points [thus: $0, 0.1, 0.2, 0.3....$etc.]. We then round the Bils-Klenow reset probabilities to 0.1 percentage point, and allocate the 300 BK sectors to these 10 focal points. The sectors are scaled by the share in expenditure that is allocated to each focal point. The resulting distribution is plotted in Figure 1. The mean frequency of price adjustment $(1 - \theta)$ across the whole economy is around 0.4. As the figure shows, there are quite a few flexible contracts. The share of flexible contacts is around 35%. But the distribution has a long tail. In the Calvo model $\theta_i = \theta$.

3 Results

We start by reproducing what is already known from prior work, that as the trend inflation rate increases, the determinacy region is shrunk in a single sector Calvo model. The determinacy region is the two dimensional space defined by the parameters on the inflation rate and the output gap in a Taylor rule for which the model is determinate. Figure 2 plots the determinacy regions for different rates of trend inflation for the Calvo model, which is a special case of our model when all the sectors face the same probability of price adjustment.

At zero inflation, the case typically covered, the determinacy region is large. All the area to the right of the almost vertical line beginning at ($\phi_\pi = 1$, $\phi_y = 0$) and heading 'North East' are determinate. As we increase the inflation rate, the region shrinks. The shrinkage can be seen by the gradual clockwise rotation of the line separating determinacy (below and to the right) from indeterminacy (above and to the left). For positive trend inflation, the determinacy borders all slope upwards, implying that greater and greater responses to inflation can 'buy' a higher value of the output gap response coefficient for which the model is still determinate. The slope of the determinacy border falls with each increment to trend inflation, implying that if we increase the output gap coefficient, we have to increase the inflation coefficient by more if we want to preserve determinacy.

However, the above findings do not provide a strong case against the policy proposal to increase the inflation target to 4%, or suggest that such a policy in the past might have led to indeterminacy prior to the era of explicit inflation targeting. As Figure 2 shows, at 4%, the determinacy region is quite large. For example, if the inflation target is increased to 4%, the central bank can achieve determinacy by simply following the Taylor rule, with coefficients $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$.

We now illustrate the effect of heterogeneity in price stickiness by recomputing these regions for simple 2-sector MCs, which preserves the mean probability of price reset in the standard 1-sector model, but in which now the two sectors have different reset probabilities. We consider two different calibrations. In one of the calibrations, we assume $\theta_1 = 0$, $\theta_2 = 0.74$ and $\alpha_1 = \alpha_2 = 0.5$. In the other, we assume $\theta_1 = 0.44$, $\theta_2 = 0.9$, $\alpha_1 = 0.95$ and $\alpha_2 = 0.05$. Figure 4 reports the resulting determinacy regions for the first calibration and 5 for the second.

The striking result from Figures 4 and 5 is that heterogeneity in price stickiness significantly magnifies the effect of trend inflation on the determinacy region. As we raise the trend inflation rate by the same increments of 2 percentage points, as before, the determinacy region is markedly smaller. This is confirmed by noting that the determinacy border is tilted clockwise relative to the same border for the single sector Calvo model above.

Having illustrated the influence of heterogeneity in 2-sector MCs, we now turn to present a more realistic calibration using a MC model that is calibrated based on the Bils-Klenow distribution (which we label BK-MC). As noted above, the BK-MC model is calibrated to have the same mean probability of price reset as the standard Calvo model. Figure 4 plots the determinacy regions for the BK-MC. In the BK-MC, increments of trend inflation decrease the determinacy region quite dramatically. In fact, at 4%, the determinacy region is almost non-existent in the BK-MC model.

Why is the determinacy region smaller in the MC model? To provide an answer to this question, we go through both the short and the long run properties of inflation.

3.1 Short-run inflation in the MC model

Allowing trend inflation in the MC leads to richer inflation dynamics. Inflation in sector i is given by

$$\widehat{\pi}_{it} = \beta\widehat{\pi}_{it+1} + \kappa_i\left(\widehat{MC}_t - \widehat{p}_{it}\right) + \beta\left(\pi - 1\right)\left(1 - \theta_i\pi^{\varepsilon-1}\right)\left(\varepsilon\widehat{\pi}_{t+1} + \widehat{\psi}_{it+1}\right) \tag{20}$$

$$= d_i\beta\widehat{\pi}_{it+1} + \kappa_i\left(\widehat{MC}_t - \widehat{p}_{it}\right) + \beta\left(\pi - 1\right)\left(1 - \theta_i\pi^{\varepsilon-1}\right)\left(\varepsilon\left(1 - \alpha_i\right)\widehat{\pi}^r_{it+1} + \widehat{\psi}_{it+1}\right) \tag{21}$$

with

$$\kappa_i = \frac{\left(1 - \theta_i\pi^{\varepsilon-1}\right)\left(1 - \theta_i\beta\pi^{\varepsilon}\right)}{\theta_i\pi^{\varepsilon-1}} \tag{22}$$

$$d_i = \left(1 + \varepsilon\left(\pi - 1\right)\left(1 - \theta_i\pi^{\varepsilon-1}\right)\right) \tag{23}$$

Assuming $\pi = 1$ gives sectoral inflation rates in the MC model without trend inflation, as in Woodford (2003, p. 203). Equation (21) uses the fact that $\widehat{\pi}_t = \sum_{i=1}^{N}\alpha_i\widehat{\pi}_{it}$. $E_t\widehat{\pi}^r_{it+1}$ denotes inflation expectations in the rest of the economy (excluding sector i). d_i is the coefficient on expected inflation in sector i, while κ_i is the coefficient on marginal costs and relative prices in sector i. Droping all i's and assuimg that $\widehat{p}_t = 0$, we obtain the Phillips curve in the Calvo model, as in Ascari and Sbordone (2014)

$$\widehat{\pi}_t = d\beta\widehat{\pi}_{t+1} + \kappa\widehat{MC}_t + \beta(\pi - 1)\left(1 - \theta\pi^{\varepsilon-1}\right)\widehat{\psi}_{t+1} \tag{24}$$

In the Calvo model, trend inflation affects all firms in the same way. An increase in trend inflation makes firms more forward-looking. Indeed, in Equation (24), the coefficient on expected inflation (d) increases with trend inflation, while the one on the output gap (κ) decreases. Since prices are sticky, firms adjust their prices more in order to protect their real prices against future inflation.

In the MC, trend inflation affects firms in different sectors differently. While firms in relatively flexible-price sector are not affected much from trend inflation, those in sticky sectors are affected significantly. Indeed, $d-$coefficients and $\kappa-$coefficients in Equations (20) and (24) depend crucially on the degree of price stickiness in sectors. Firms in sectors that have contracts longer than average contract length are more influenced by trend inflation than those in the Calvo model.

Another implication of trend inflation is that, as shown by Schmitt-Grohe and Uribe (2007), positive trend inflation creates a wedge between labour supply and output, resulting in a lower output for a given labour supply. To put it differently, to produce the same level of output in a positive trend inflation environment more labour input is required. This can be easily seen by combining equations (1) and (4) and aggregating across firms. The resulting equation is

$$Y_t = \frac{N_t A_t}{s_t} \tag{25}$$

When trend inflation is positive, $s_t > 1$.Increased labour demand leads to an increase in wages and, therefore, marginal cost. Trend inflation leads to larger price dispersion, higher marginal cost, higher inflation and lower output. When we assume $\varphi = 0$ (i.e. indivisible labour) and there is no productivity shock, s_t does not matter. $\widehat{MC}_t$ solely depends on output $\widehat{MC}_t = \sigma\hat{Y}_t$.

While all these are true in the single sector model, this channel (when $s_t > 1$) is stronger in the MC model since, because of the presence of heterogeneity in price stickiness, price dispersion is larger. Our results holds true even when we assume $\varphi = 0$.

Finally, it is interesting to note that, sectoral inflation rates depend not only on sector-specific expected inflation but also expected aggregate inflation. To understand why this is the case first note that, while the sectors with relatively flexible prices do not need to worry about the future as much as the sectors with sticky prices, for sticky sectors, the future becomes increasingly more important with trend inflation. To preserve relative prices, firms in the sectors with relatively flexible prices become more forward-looking, too.

3.2 Long-run inflation in the MC model

The long-run or the steady-state of the MC model is significantly affected by trend inflation. The main effect is that price mark-up in different sectors increase with trend inflation. To see this, note that the steady state version of Equation (6)

$$p_i^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{j=0}^{\infty} (\beta\theta_i\pi^{\varepsilon})^j MC}{\sum_{j=0}^{\infty} (\beta\theta_i\pi^{\varepsilon-1})^j} \tag{26}$$

Therefore, price mark-up $\left(\frac{p_i^*}{MC}\right)$ in sector i is given by

$$\frac{p_i^*}{MC} = \frac{\varepsilon}{\varepsilon - 1} \frac{1 - \beta\theta_i\pi^{\varepsilon-1}}{1 - \beta\theta_i\pi^{\varepsilon}} \tag{27}$$

This equation clearly shows that mark-up in sector i increases with trend inflation, implying steady-state output is different in different sectors. As trend inflation increases, steady-state output decreases more in sectors with longer-term contacts. The reason for this result is that firms in sectors with longer-term contacts increase their prices more aggressively to protect their relative prices from inflation. In the special case in which steady-state inflation rate is zero, the degree of mark-up is the same across sectors and is given by $\frac{\varepsilon}{\varepsilon-1}$. In the Calvo model, since $\theta_i = \theta$, while mark-up increases with trend inflation, the degree of price stickiness is the same across sectors.

This insight suggests that increased inflation leads to lower steady-state output. This can easily be shown by considering the long-run relationship between inflation and output in the 2-sector version of the MC, where in sector 1 prices are fully flexible, while it is sticky in sector 2. To derive this relationship, we made two simplifying assumptions: $\varphi = 0$ and $\sigma = 1$. In the long-run, the inflation rate is the same across sectors ($\hat{\pi}_i = \pi$), $\hat{Y} = Y$ and $\widehat{\psi}_2 = \psi_2$. Using this and the fact that $-\widehat{p}_{2t} = \frac{\alpha_1}{\alpha_2}\widehat{p}_{1t} = \frac{\alpha_1}{\alpha_2}\hat{Y}_t$, we obtain the following long-run relationship between inflation and output

$$\pi = \beta\pi + \frac{\kappa_2}{\alpha_2}Y + \beta(\pi - 1)\left(1 - \theta_2\pi^{\varepsilon-1}\right)(\varepsilon\pi + \psi_2) \qquad (28)$$

with

$$\psi_2 = Y + \frac{\beta\theta_2\pi^{\varepsilon}\varepsilon}{(1 - \beta\theta_2\pi^{\varepsilon})}\pi \qquad (29)$$

After a straightforward algebra, we first express output in terms of aggregate inflation and then take the first derivative of output with respect to inflation, which is given by

$$\frac{dY}{d\pi}\mid_{LR} = \frac{1 - \beta\left(1 - (\pi - 1)(1 - \theta_2\pi^{\varepsilon-1})\frac{\varepsilon}{(1-\beta\theta_2\pi^{\varepsilon})}\right)}{\frac{\kappa_2}{\alpha_2} + \beta(\pi - 1)(1 - \theta_2\pi^{\varepsilon-1})} \qquad (30)$$

Figure 6 plots the value of this derivative against trend inflation both in the Calvo and in the MC. Let us first consider the Calvo case. Assuming $\alpha_2 = 1$, $\theta_2 = \theta$ and $\kappa_2 = \kappa$ gives the Calvo model. In the Calvo model without trend inflation ($\pi = 1$), $\frac{\partial\hat{Y}}{\partial\hat{\pi}}\mid_{LR}$reduces to the familiar result $(1 - \beta)/\kappa$ (see Woodford (2003, p. 254)). Since $0 < \beta < 1$ and, with plausible parameter values, $\kappa > 0$, the derivative is positive but is close to zero.

With trend inflation, this derivative is negative. The second term in the numerator can be thought as the discount factor in the presence of positive trend inflation. As higher trend inflation makes firms more forward-looking, the second term in the numerator becomes greater (>1), resulting in a negative numerator. Increased forward-lookingness also lowers κ_2, increasing the absolute value of the derivative.

We now turn to the MC. As Figure 6 shows that the value of the derivative is slightly positive when trend inflation is low but becomes increasingly negative, as trend inflation increases. Even at a low level of trend inflation, in the MC, it is more negative than in the Calvo model. The difference in results is easy to understand. In the MC, there are longer-term contracts, meaning that some firms are more forward-looking than those in the Calvo model. As a result, discount factor in the MC is higher and κ_2 is lower, leading to a more negative slope of the Phillips curve.

As an aside, notice that the always negative slope of $\frac{dY}{d\pi} \mid_{LR}$implies that in the MC model the total output costs of a given increase in the inflation target above zero will be greater than with the single sector model. (This total cost will be related to the integral under the curve in Figure 6). This provides an additional reason - aside from the effect on determinacy - why the single sector model understates the cost of raising the inflation target relative to the benefit (in terms of avoiding the liquidity trap).

3.3 Indeterminacy region in the MC

The effects of these changes on the determinacy region can be understood by using Woodford's (2003) interpretation of the Taylor principle. In Woodford's interpretation the emphasis is on the long-run. As noted above, to achieve determinacy, in the long-run, the nominal interest rate should increase by more than the increase in inflation. This definition implies that equilibrium is determinate if

$$\frac{\partial i}{\partial \pi} \mid_{LR} = \phi_\pi + \phi_y \frac{\partial Y}{\partial \pi} \mid_{LR} > 1 \tag{31}$$

As discussed above, in the Calvo model, $\frac{\partial Y}{\partial \pi} \mid_{LR}$is close to zero but positive. As a result, given the condition that $\phi_\pi > 1 - \phi_y \frac{\partial Y}{\partial \pi} \mid_{LR}$, equilibrium can be determinate even when $\phi_\pi < 1$. When trend inflation is positive, we obtain the result reported in Ascari. Ascari shows that $\frac{\partial Y}{\partial \pi} \mid_{LR}$is negative, as trend inflation increases. As a result, ϕ_π should increase with trend inflation to achieve determinacy. In the MC, since $\frac{\partial Y}{\partial \pi} \mid_{LR}$is even more negative, ϕ_π must be larger in the MC than in the Calvo model. Thus, the presence of longer-term contracts in the MC leads to smaller determinacy region.

4 Conclusions

The prolonged period spent by many central banks at the zero bound naturally leads one to ask whether, supposing that an escape from the liquidity trap can successfully be fashioned, it would be better in future to target a higher inflation rate, so as to lower the chance of repeating this experience. This paper articulates one reason why such a policy might be cautioned against.

We take a version of the New Keynesian model that has two modifications relative to the standard version. First, it is linearised around positive steady-state rates of inflation.Second, we allow the degree of price stickiness to vary across sectors, encoding observations made in Bils and Klenow (2004) and others. We use this model to study how the region for which monetary policy rules render rational expectations equilibria indeterminate is enlarged as the inflation rate increases. This phenomenon had already been noted by in an otherwise standard representative firm version of the New Keynesian model. Our contribution, therefore, is to revisit that work in model that accounts for heterogeneity in prices stickiness we have observed in the micro-data.

We find that the conclusion by Ascari and Sbordone (2014) that the indeterminacy region grows as target inflation raises survives, but that the indeterminacy region grows a great deal more as target inflation rises in the heterogeneous price stickiness (MC) version of the model. At the commonly proposed target of 4%, in a multi-sector model, the indeterminacy region is very small. In the corresponding standard Calvo model, however, the determinacy region is quite large and is not too different from the region when the target is 2%. In the standard model with a 4% inflation target, the equilibrium is determinate even when the central bank follows the Taylor rule, with the commonly assumed coefficients $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$.

It has been noted that the determinacy region shrinks with increasing trend inflation, since the slope of the Phillips curve becomes more negative with increasing trend inflation. In the MC, the slope of the Phillips curve becomes even more negative. This is due to the fact that price mark-ups in the model becomes larger in the sticky-prices sectors when trend inflation increases.

As well as heightening concern about the option of a 4% target over the future, a corollary of our work is that historically high inflation rates in economies like the US and the UK are much more likely to have led to indeterminacy than researchers may previously have been aware. This lends credence to the argument that sunspot shocks to inflation - the possibility for which is opened up by indeterminacy - were part of the explanation for past inflation volatility.

Finally, an interesting side-product emerges from the analysis. Even firms in sectors with relatively flexible prices respond in a forward-looking manner. This is because of the fact firms in sticky sectors make larger adjustment, as trend inflation increases, since they have to charge the same price for a long time. To preserve relative prices, firms in the sectors with flexible prices becomes forward-looking too. In the future work, we plan to investigate the empirical relevance of this new channel using both aggregate macro data and sectoral price data.

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Appendix

As noted in the main text, the unit of firms, indexed by $f\epsilon\,[0,1]$ is divided into sectors. There are N sectors, $i = 1...N$. Sectors has different Calvo hazard rates $(1-\theta_i)$ and in their sector shares (α_i). Firm f in sector i solves the following profit maximisation problem:

$$E_t\sum_{j=0}^{\infty}\beta^j\frac{\lambda_{t+j}}{\lambda_0}\theta_i^j\left[\frac{P_{fit}^*}{P_{t+j}}Y_{fit+j}-\frac{W_{t+j}}{P_{t+j}}\frac{Y_{fit+j}}{A_{t+j}}\right] \tag{32}$$

subject to the demand function faced by a firm f in sector i

$$Y_{fit+j}=\left[\frac{P_{fit}^*}{P_{t+j}}\right]^{-\varepsilon}Y_{t+j} \tag{33}$$

Substituting equation (33) into (32) and solving the maximisation problem, we obtain

$$\frac{P_{it}^*}{P_t}=\frac{(-\varepsilon)}{(1-\varepsilon)}\frac{E_t\sum_{j=0}^{\infty}\beta^j\lambda_{t+j}\theta_i^j w_{t+j}\left[\frac{1}{\Pi_{t,t+j}}\right]^{-\varepsilon}}{E_t\sum_{j=0}^{\infty}\beta^j\lambda_{t+j}\theta_i^j\left[\frac{1}{\Pi_{t,t+j}}\right]^{1-\varepsilon}} \tag{34}$$

where $\Pi_{t,t+j}=\frac{P_{t+j}}{Pt}$, $\lambda_{t+j}=C_{t+j}^{-\sigma}=Y_{t+j}^{-\sigma}$ and $w_{t+j}=\frac{W_{t+j}}{P_{t+j}}$. Note that subscript f is dropped in the above equation, as all the firms that reset their prices in sector i set the same price. Define ψ_{it} and ϕ_{it}

$$\psi_{it}=E_t\sum_{j=0}^{\infty}\beta^j\lambda_{t+j}\theta_i^j\frac{W_{t+j}}{P_{t+j}}\left[\frac{1}{\Pi_{t,t+j}}\right]^{-\varepsilon}Y_{t+j}$$
$$\phi_{it}=E_t\sum_{j=0}^{\infty}\beta^j\lambda_{t+j}\theta_i^j\left[\frac{1}{\Pi_{t,t+j}}\right]^{1-\varepsilon}Y_{t+j}$$

ψ_{it} and ϕ_{it} can be rewritten recursively as follows

$$\phi_{it} = \beta\theta_i \pi_{t+1}^{\varepsilon-1}\phi_{it+1}$$
$$\psi_{it} = A_t^{-1} w_t + \beta\theta_i \pi_t^{\varepsilon}\psi_{it+1}$$

Log-linearsing these equations along with Equation (34) gives Equations (9)-(11) in the main text. The average price level in sector i is

$$\frac{P_{it}}{P_t} = \left[\theta_i \left(\frac{P_{it-1}}{P_{t-1}}\frac{P_{t-1}}{P_t}\right)^{1-\varepsilon} + (1-\theta_i)\left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{35}$$

The Log-linearsed version of this equation is reported in Equation (12) in the main text. To calculate price dispersion, we use the production function (Equation (1)). Aggregating over firms within the same sector, we obtain

$$N_{it} = \frac{Y_{it}}{A_t}$$

Substituting Equation (4) into this equation and aggregating across sectors gives

$$N_t = \sum_{i=1}^{N} \alpha_i \frac{Y_{it}}{A_t} = \frac{Y_t}{A_t} \underbrace{\sum_{i=1}^{N} \alpha_i \left[\frac{P_{it}}{P_t}\right]^{-\varepsilon}}_{s_{it}}$$

where s_{it} is the relative price dispersion measure in sector i and captures the cost of relative price dispersion in that sector due to positive trend inflation. This measure can be rewritten as

$$s_{it} = (1-\theta_i)\left[\frac{P^*_{it}}{P_t}\right]^{-\varepsilon} + \theta_i(1-\theta_i)\left[\frac{P^*_{it-1}}{P_t}\right]^{-\varepsilon} + \theta_i^2(1-\theta_i)\left[\frac{P^*_{it-2}}{P_t}\right]^{-\varepsilon} + ... \tag{36}$$

$$= (1-\theta_i)\left[\frac{P^*_{it}}{P_t}\right]^{-\varepsilon} + \theta_i(1-\theta_i)\left[\frac{P^*_{it-1}}{P_{t-1}}\frac{P_{t-1}}{P_t}\right]^{-\varepsilon} + \theta_i^2(1-\theta_i)\left[\frac{P^*_{i,t-2}}{P_{t-2}}\frac{P_{t-2}}{P_{t-1}}\frac{P_{t-1}}{P_t}\right]^{-\varepsilon} + ... \tag{37}$$

$$s_{it} = (1-\theta_i)\left[\frac{P^*_{it}}{P_t}\right]^{-\varepsilon} + \theta_i\pi_t^{\varepsilon}s_{it-1} \tag{38}$$

Equation (13) reports the log-linearised version of Equation (38)

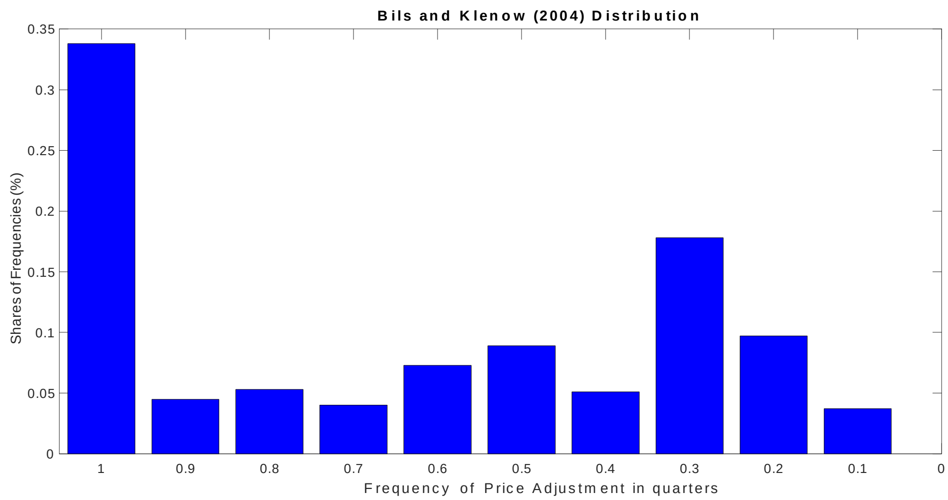

Figure 1: The Bils and Klenow (2004) distribution of price spells
Note: Using the US CPI data, Bils and Klenow (2004) report the frequency of price changes for around 300 product categories, which covers 70% of the US CPI. These frequencies are rounded to one decimal point and resuting numbers aggregated up so, leading to 10 distinct price reset probabilities, which are reported in this figure.

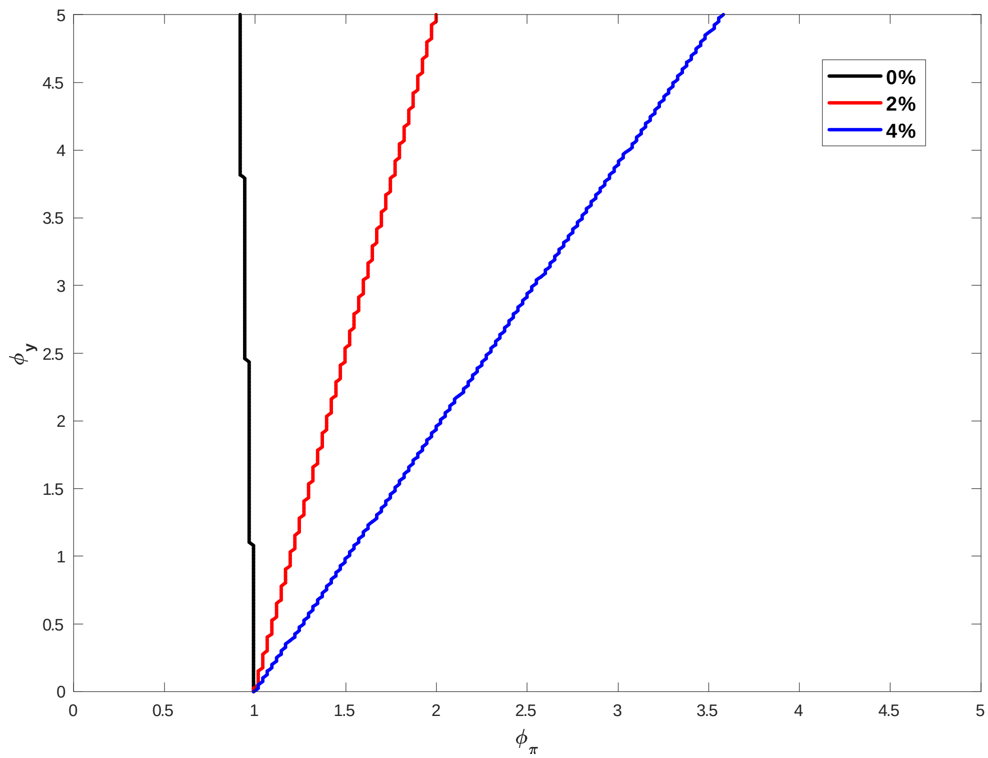

Figure 2: The determinancy regions for alternative trend inflation rates in the standard Calvo model
Note: Consistent with the findings reported in Ascari and Sbordone (2004), this figure shows that increasing trend inflation shrinks the determinancy region.

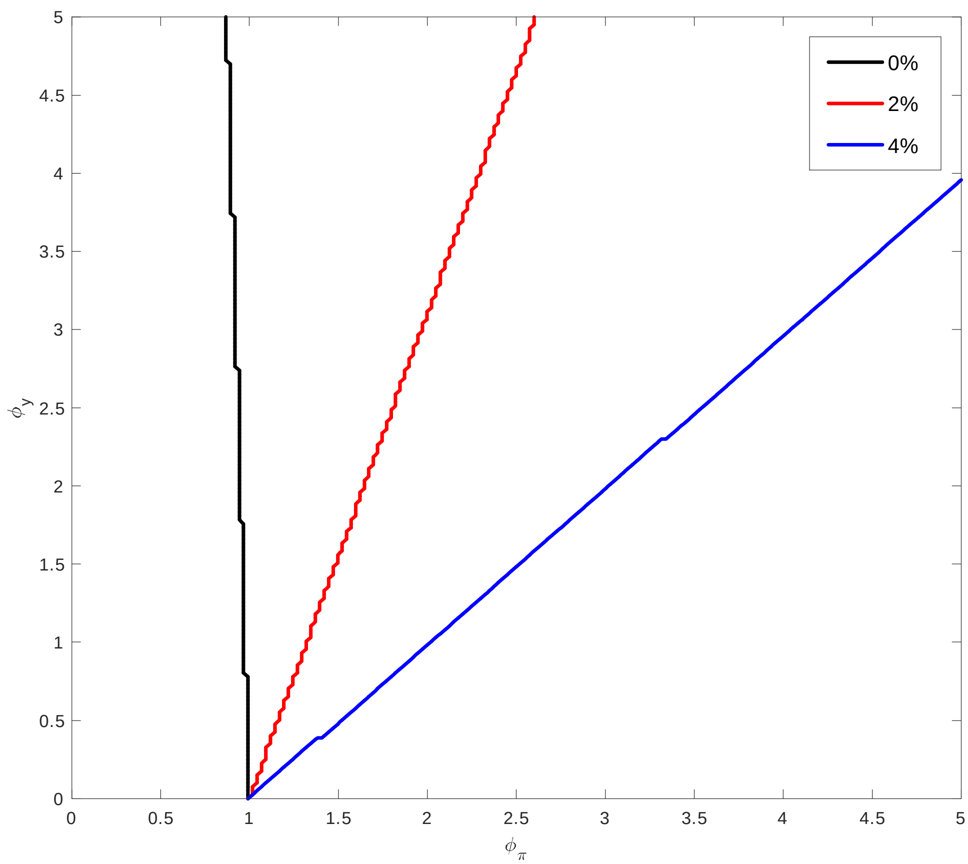

Figure 3: The determinancy regions for alternative trend inflation rates in the MC with $\theta_1 = 0$, $\theta_2 = 0.74$ and $\alpha_1 = \alpha_2 = 0.5$. Notes: The mean contract length in this model is the same as that in the Calvo model in Figure 2.

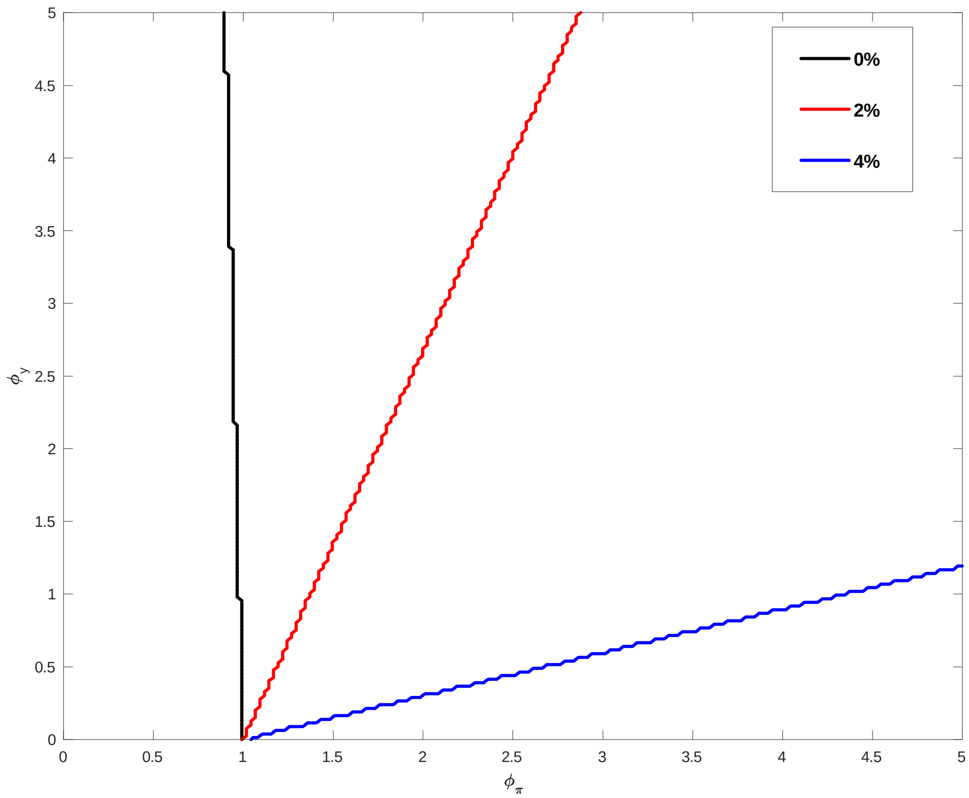

Figure 4: The determinancy regions for alternative trend inflation rates in the MC with $\theta_1 = 0.44$, $\theta_2 = 0.9$, $\alpha_1 = 0.95$ and $\alpha_2 = 0.05$.
Notes: The mean contract length in this model is the same as that in the Calvo model in Figure 2.

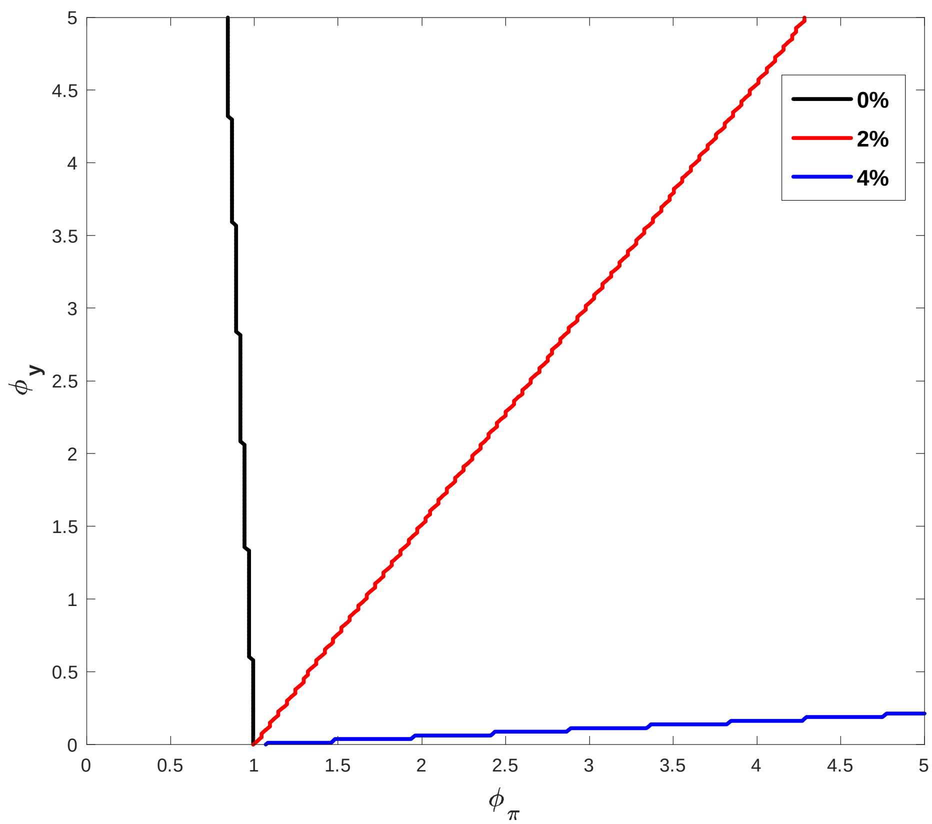

Figure 5: The determinancy regions in the MC with Bils and Klenow (2004) distribution model for alternative trend inflation rates
Notes: The mean contract length in this model is the same as that in the Calvo model in Figure 2.

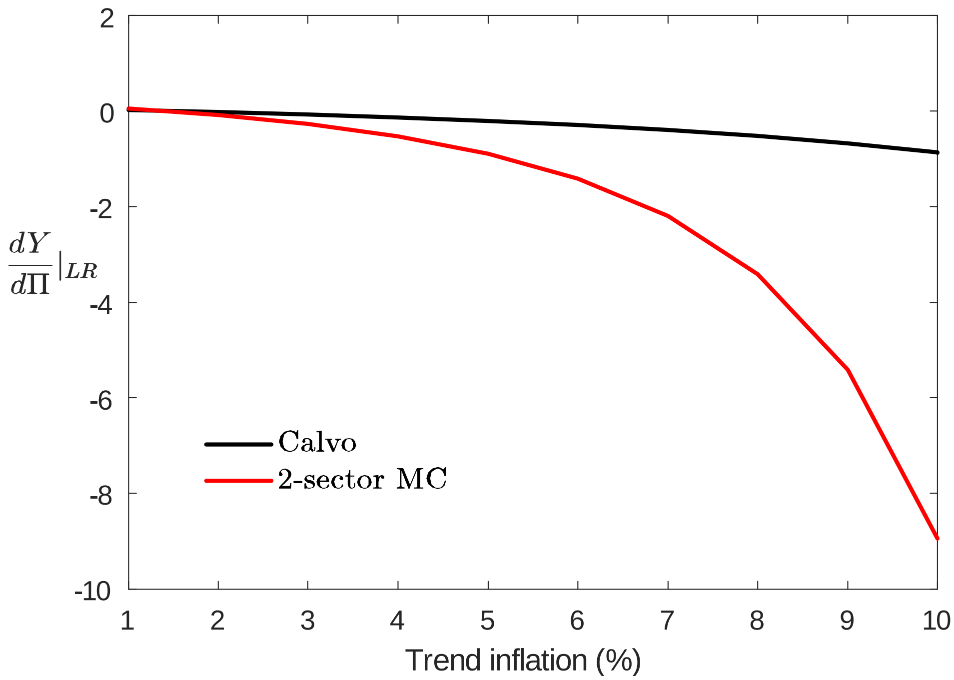

Figure 6: The slope of the Phillips curve in the long run and trend inflation

Notes: The slope of the Phillips curve becomes more negative in the MC than in the Calvo, as trend inflation increases.